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Symplectic structure-free Chern–Simons theory

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Abstract. The second-class constraints algebra of the Abelian Chern–Simons (CS) theory is studied rigorously in terms of Hamiltonian embedding in order to obtain a first-class constraint system. The well known symplectic structure of the CS fields due to the second-class constraints disappears in the resulting system. We then obtain a new type of CS action which has an infinite set of irreducible first-class constraints and exhibits new extended local gauge symmetries implemented by these first-class constraints.

Chern–Simons (CS) theories [1] have been rigorously studied in various arenas. One of the intriguing problems of CS theories is that the CS Lagrangian, from the point of view of a constrained system, gives the unusual second-class constraints even though it is invariant up to a total divergence under the local gauge transformation. This peculiar property of CS theories is essentially due to the symplectic structure [2] which is a key ingredient of CS systems. Meanwhile, the second-class constraint system has been generically regarded as a gauge fixed version of the gauge invariant system [3], which has been studied in the context of anomalous gauge theory [4]. Therefore, CS theories may have some kind of unknown local symmetry if second-class constraints are converted into first-class ones. This means that the intrinsic symplectic structure of CS theories can be interpreted as a gauge fixed form of gauge invariant theory which is symplectic-free. What then is the additional local symmetry in connection with the symplectic structure?

It is, in general, difficult to convert the second-class constraint system even for the Abelian-pure CS theory into a first-class one by using the usual Stückelberg mechanism [5] since the origin of the second-class constraints algebra is unusual compared to the conventional anomalous theory. On the other hand, Batalin, Fradkin and Tyutin (BFT) Hamiltonian embedding of a model [8] is very useful and it converts systematically the second-class system into a first-class one. According to the usual treatment [6, 7, 9–11] of the BFT formalism, one simply identifies auxiliary fields with a pair of conjugate fields. This procedure in CS theory, however, gives an undesirable final expression [6, 10] in that the original action has not been reproduced when we choose the unitary gauge [6, 7] and the additional action so-called Wess–Zumino (WZ) action, which is needed to make the system gauge invariant, is model-dependent. Furthermore, the assumed brackets of the auxiliary fields are not Poisson but Dirac brackets.

In this paper by introducing infinite auxiliary fields, we find a new type of WZ action for the Abelian-pure CS theory so that the total system has fully first-class constraints which have not been successful so far. This total action is naturally reduced to the original CS action if one chooses unitary gauge conditions. Then, we obtain new symmetries corresponding to first-class constraints relating to the symplectic structure as well as the well known local U(1) gauge symmetry.

Let us start with the Abelian-pure CS Lagrangian

$$\mathcal{L}_0 = \frac{\kappa}{2}\epsilon_{\mu\nu\rho}A^{\mu}\partial^{\nu}A^{\rho}. \tag{1}$$

The canonical momenta are given by $\pi_0 = 0$ and $\pi_i = (\kappa/2)\epsilon_{ij}A^j$. We then have three primary constraints [12], $\Omega_0 = \pi_0 \approx 0$, $\Omega_i = \pi_i - (\kappa/2)\epsilon_{ij}A^j \approx 0$ $(i, j = 1, 2)$ and a secondary constraint

$$\Omega_3 = \kappa\epsilon_{ij}\partial^i A^j \approx 0 \tag{2}$$

which is obtained from the stability condition of time evolution of the constraint Ω_0 with the primary Hamiltonian $H_{\rm p} = H_{\rm c} + \int {\rm d}^2x(v^0\Omega_0 + v^i\Omega_i)$. The canonical Hamiltonian is given by $H_{\rm c} = \int {\rm d}^2x\,\mathcal{H}_{\rm c} = \int {\rm d}^2x\,\kappa A^0\epsilon^{ij}\partial_i A^j$. No additional constraints are generated from the consistency conditions of the other constraints Ω_i and Ω_3 by fixing the Lagrange multipliers as $v^i = \kappa\partial^i A^0$.

To obtain the maximally irreducible first-class constraints [6, 10], we redefine the primary and secondary constraints as $\omega_0 \equiv \Omega_0$, $\omega_i \equiv \Omega_i$ and

$$\omega_3 \equiv \Omega_3 + \partial^i\Omega_i = \partial^i\pi_i + \frac{\kappa}{2}\epsilon_{ij}\partial^i A^j \approx 0. \tag{3}$$

Eliminating the Lagrange multipliers v^i yields the extended Hamiltonian density [13] corresponding to the CS Lagrangian

$$\mathcal{H}_{\rm E} = v^0\omega_0 - (A^0 - v^3)\omega_3 \tag{4}$$

where the Lagrange multipliers v^0 and v^3 remain undetermined. The extended Hamiltonian now naturally generates the Gauss constraint ω_3 from the time evolution of the constraint ω_0 as

$$\dot{\omega}_0 = \omega_3 \qquad \dot{\omega}_\alpha = 0 \qquad \alpha = 1, 2, 3 \tag{5}$$

where overdot represents the time evolution. We, therefore, have two first-class constraints ω_0 and ω_3 and two second-class constraints ω_i which satisfy the constraint algebra

$$\Delta_{ij}(x, y) \equiv \{\omega_i(x), \omega_j(y)\} = -\kappa\epsilon_{ij}\delta(x - y). \tag{6}$$

Upon elimination of the momenta π_i via the method of Dirac [12], we could easily obtain the well known Dirac brackets for the gauge fields A^i as $\{A^i(x), A^j(y)\} = \epsilon^{ij}\delta(x - y)/\kappa$.

Compared to this phase space reduction, one can embed a second-class structure into a first-class one by introducing auxiliary fields in BFT Hamiltonian embedding [8]. In order to explicitly make the analysis, let us first rewrite the CS Lagrangian replacing A^μ with $A^{(0)\mu}$ as

$$\mathcal{L}_0 \equiv \mathcal{L}^{(0)} = \frac{\kappa}{2}\epsilon_{\mu\nu\rho}A^{(0)\mu}\partial^{\nu}A^{(0)\rho}. \tag{7}$$

We now introduce auxiliary fields $A^{(1)i}$ to make the second-class constraints ω_i into first-class ones satisfying $\{A^{(1)i}(x), A^{(1)j}(y)\} = \vartheta^{ij}(x, y)$. Making use of the auxiliary fields $A^{(1)i}$, we could write the effective first-class constraints as $\tilde{\omega}_i(\pi^{(0)}_\mu, A^{(0)\mu}; A^{(1)i}) = \omega_i + \sum_n \varpi_i^{(n)}$ satisfying the boundary condition $\tilde{\omega}_i(\pi^{(0)}_\mu, A^{(0)\mu}; 0) = \omega_i$ as well as requiring the strong involution, i.e. $\{\tilde{\omega}_i, \tilde{\omega}_j\} = 0$. Here $\varpi_i^{(n)}$ is assumed to be proportional to $(A^{(1)i})^n$. In particular, the first-order correction in these infinite series is given by

$$\varpi_i^{(1)} = \int {\rm d}^2y\, X_{ij}(x, y)A^{(1)j}(y) \tag{8}$$

and the requirement of the strong involution gives the following condition

$$\Delta_{ij}(x,y) + \int \mathrm{d}^2u\, \mathrm{d}^2v\, X_{ik}(x,u)\vartheta^{k\ell}(u,v)X_{j\ell}(v,y) = 0. \tag{9}$$

We take the simple solution of ϑ^{ij} and X_{ij} as

$$\vartheta^{ij}(x,y) = \epsilon^{ij}\delta(x-y) \tag{10}$$

$$X_{ij}(x,y) = -\epsilon_{ij}\delta(x-y)/\sqrt{\kappa}. \tag{11}$$

There is some arbitrariness in choosing ϑ^{ij} and X_{ij} from (9) as shown in the literature [6, 7, 9], which is related to canonical transformation. For the choice of (10), equation (9) is simply reduced to $|\det X_{ij}| = 1/\kappa$ and a convenient solution is given as equation (11).

By using ϑ^{ij} and the solution of X_{ij} in equations (10) and (11), we obtain the strongly involutive first-class constraints which are only proportional to the first order of the auxiliary fields, as

$$\tilde{\omega}_i^{(0)} = \pi_i^{(0)} - \frac{\kappa}{2}\epsilon_{ij}A^{(0)j} - \sqrt{\kappa}\epsilon_{ij}A^{(1)j} = 0. \tag{12}$$

The canonical Hamiltonian density is now given by

$$\tilde{\mathcal{H}}_{\mathrm{c}} = \kappa A^{(0)0}\epsilon^{ij}\partial_i\left(A^{(0)j} + \frac{1}{\sqrt{\kappa}}A^{(1)j}\right) \tag{13}$$

satisfying $\{\tilde{\omega}_i, \tilde{H}_{\mathrm{c}}\} = 0$ and $\{\omega_0, \tilde{H}_{\mathrm{c}}\} = \{\omega_3, \tilde{H}_{\mathrm{c}}\} = 0$. The corresponding Lagrangian of equation (13) with two auxiliary fields $A^{(1)i}$ is obtained through the usual path integral as follows

$$\mathcal{L}^{(1)} = -\frac{\kappa}{2}\epsilon_{ij}A^{(0)i}\dot{A}^{(0)j} + \kappa A^{(0)0}\epsilon_{ij}\partial^i A^{(0)j} - \frac{1}{2}\epsilon_{ij}A^{(1)i}\dot{A}^{(1)j} + \sqrt{\kappa}A^{(0)0}\epsilon_{ij}\partial^i A^{(1)j} - \sqrt{\kappa}\epsilon_{ij}A^{(1)i}\dot{A}^{(0)j}. \tag{14}$$

In the BFT Hamiltonian embedding of the model one has usually identified two auxiliary fields $A^{(1)i}$ with a pair of conjugate fields as a coordinate and momentum [6, 7, 9–11]. However, we would like to note that there is no general preference to choose them as the conjugate fields.

To clarify this problem, let us now study whether or not the Lagrangian (14) produces a first-class constraint system at the Poisson bracket level which is an essential element of the BFT formalism. The canonical momenta from (14) are $\pi_0^{(0)} = 0$, $\pi_i^{(0)} = (\kappa/2)\epsilon_{ij}A^{(0)j} + \sqrt{\kappa}\epsilon_{ij}A^{(1)j}$ and $\pi_i^{(1)} = (1/2)\epsilon_{ij}A^{(1)j}$. From the conditions of time stability of these primary constraints, we can get one more secondary constraint and after redefining we can easily obtain the maximally irreducible first-class constraints as $\omega_0 = \pi_0^{(0)} \approx 0$, $\omega_3 = \partial^i\pi_i^{(0)} + (\kappa/2)\epsilon_{ij}\partial^i A^{(0)j} \approx 0$ and

$$\tilde{\omega}_i^{(1)} = \pi_i^{(0)} - \frac{\kappa}{2}\epsilon_{ij}A^{(0)j} - \sqrt{\kappa}\left(\pi_i^{(1)} + \frac{1}{2}\epsilon_{ij}A^{(1)j}\right) \approx 0 \tag{15}$$

as well as the second-class constraints

$$\omega_i^{(1)} = \pi_i^{(1)} - \tfrac{1}{2}\epsilon_{ij}A^{(1)j} \approx 0. \tag{16}$$

Therefore, second-class constraints remain even after the first order of correction. On the other hand, we can calculate the (preliminary) Dirac brackets as

$$\{A^{(1)i}, A^{(1)j}\}_D = \epsilon^{ij}\delta(x-y) \tag{17}$$

which is the relation introduced in equation (10) to make the second-class constraints ω_i into first-class ones $\tilde{\omega}_i^{(0)}$ in the BFT formalism, i.e. the first-class constraints $\tilde{\omega}_i^{(1)}$ reduce to $\tilde{\omega}_i^{(0)}$ if we strongly impose the second-class constraints (16). As a result we observe that the auxiliary fields $A^{(1)i}$ introduced to make second-class constraints into first-class ones do not provide

Poisson but Dirac bracket structure. A similar feature appeared in chiral boson theory [14, 15] and recently in string and D-branes theory [16]. This makes the BFT Hamiltonian embedding of CS theory not to stop any finite number of steps. Therefore, these steps should infinitely be repeated and thus one can construct an action which has fully first-class constraints by introducing infinite auxiliary fields $A^{(n)i}$. In this respect, all previous results [6, 7, 10] of BFT formalism applied to CS cases are incomplete. Hence, the final action can be written

$$\mathcal{L} = -\frac{\kappa}{2}\epsilon_{ij}A^{(0)i}\dot{A}^{(0)j} + \kappa A^{(0)0}\epsilon_{ij}\partial^i A^{(0)j} - \frac{1}{2}\epsilon_{ij}\sum_{n=1}^{\infty}A^{(n)i}\dot{A}^{(n)j} + \sqrt{\kappa}A^{(0)}\epsilon_{ij}\sum_{n=1}^{\infty}\partial^i A^{(n)j}$$
$$-\sqrt{\kappa}\epsilon_{ij}\sum_{n=1}^{\infty}A^{(n)i}\dot{A}^{(0)j} - \epsilon_{ij}\sum_{n=1}^{\infty}\sum_{m=n+1}^{\infty}A^{(m)i}\dot{A}^{(n)j}. \tag{18}$$

To examine whether or not equation (18) really gives a first-class constraint system, we should check the constraint algebra by using Poisson brackets. The canonical momenta from (18) are given by

$$\pi_0^{(0)} = 0 \qquad \pi_i^{(0)} = \frac{\kappa}{2}\epsilon_{ij}A^{(0)j} + \sqrt{\kappa}\epsilon_{ij}\sum_{m=1}^{\infty}A^{(m)j} \qquad \pi_i^{(n)} = \frac{1}{2}\epsilon_{ij}A^{(n)j} + \epsilon_{ij}\sum_{m=n+1}^{\infty}A^{(m)j} \tag{19}$$

where $n = 1, 2, \ldots, \infty$. We, thus, have the primary constraints

$$\Omega_0 = \pi^{(0)} \approx 0$$
$$\Omega_i^{(0)} = \pi_i^{(0)} - \frac{\kappa}{2}\epsilon_{ij}A^{(0)j} - \sqrt{\kappa}\epsilon_{ij}\sum_{m=1}^{\infty}A^{(m)j} \approx 0$$
$$\Omega_i^{(n)} = \pi_i^{(n)} - \frac{1}{2}\epsilon_{ij}A^{(n)j} - \epsilon_{ij}\sum_{m=n+1}^{\infty}A^{(m)j} \approx 0 \tag{20}$$

whose conditions of time stability give only one further constraint

$$\Omega_3 \equiv \dot{\Omega}_0 = \kappa\epsilon_{ij}\partial^i\left(A^{(0)j} + \frac{1}{\sqrt{\kappa}}\sum_{m=1}^{\infty}A^{(m)j}\right) \approx 0 \tag{21}$$

with primary Hamiltonian density

$$\mathcal{H}_{\rm p} = \mathcal{H}_{\rm c} + v^0\Omega_0 + \sum_{n,i}v^{(n)i}\Omega_i^{(n)}. \tag{22}$$

The canonical Hamiltonian density is given by $\mathcal{H}_{\rm c} = \kappa A^{(0)0}\epsilon^{ij}\partial_i(A^{(0)j} + (\kappa)^{-1/2}\sum_{m=1}^{\infty}A^{(m)j})$. The maximally irreducible first-class constraints are now obtained from redefining equations (20) and (21) as $\omega_0 = \pi_0^{(0)} \approx 0$, $\omega_3 = \partial^i\pi_i^{(0)} + (\kappa/2)\epsilon_{ij}\partial^i A^{(0)j} \approx 0$ and

$$\tilde{\omega}_i^{(1)} = \pi_i^{(0)} - \frac{\kappa}{2}\epsilon_{ij}A^{(0)j} - \sqrt{\kappa}\left(\pi_i^{(1)} + \frac{1}{2}\epsilon_{ij}A^{(1)j}\right) \approx 0$$
$$\tilde{\omega}_i^{(n+1)} = \pi_i^{(n)} - \frac{1}{2}\epsilon_{ij}A^{(n)j} - \left(\pi_i^{(n+1)} + \frac{1}{2}\epsilon_{ij}A^{(n+1)j}\right) \approx 0 \tag{23}$$

where $n = 1, 2, \ldots, \infty$ and the extended Hamiltonian density has the form

$$\tilde{\mathcal{H}}_{\rm E} = \lambda^0\omega_0 - (A^{(0)0} - \lambda^3)\omega_3 + \sum_{n=1}^{\infty}\lambda^{(n)i}\tilde{\omega}_i^{(n)}. \tag{24}$$

These constraints are all involutive

$$\{\omega_0, \tilde{H}_{\rm E}\} = \omega_3 \qquad \{\omega_3, \tilde{H}_{\rm E}\} = 0 \qquad \{\tilde{\omega}_i^{(n)}, \tilde{H}_{\rm E}\} = 0. \tag{25}$$

So, the new CS theory with infinite auxiliary fields now completely forms the first-class constrained system and the strongly vanishing Poisson brackets between ω_0, ω_3 and equation (23).

It seems appropriate to comment on the constraints, ω_0, ω_3 and equation (23). The constraint ω_3 is the usual Gauss constraint related to the time-independent gauge transformation and it is not modified through BFT procedure. This reflects the maintenance of the well known original U(1) gauge symmetry. On the other hand, the infinite number of first-class constraints (23) are related to a kind of unknown local symmetry and thus the symplectic structure of the original fields can be regarded as a gauge fixed structure of modified CS theory (18).

Now we are ready to discuss new local symmetries of our first-class action. The first-order form of the action is described as

$$S = \int \mathrm{d}^3x \left(\pi_0^{(0)} \dot{A}^{(0)0} + \sum_{n=0}^{\infty} \pi_i^{(n)} \dot{A}^{(n)i} - \tilde{\mathcal{H}}_{\mathrm{E}} \right). \tag{26}$$

This action is invariant under the following gauge transformations

$$\begin{aligned}
\delta A^{(0)0} &= \epsilon^0 \\
\delta A^{(0)i} &= -\partial^i \epsilon^3 + \frac{1}{\sqrt{\kappa}} \epsilon^{(1)i} \\
\delta A^{(n)i} &= -\epsilon^{(n)i} + \epsilon^{(n+1)i} \\
\delta \pi^{(0)} &= 0 \\
\delta \pi_i^{(0)} &= -\frac{\kappa}{2} \epsilon_{ij} \partial^j \epsilon^3 - \frac{\sqrt{\kappa}}{2} \epsilon_{ij} \epsilon^{(1)j} \\
\delta \pi_i^{(n)} &= -\frac{1}{2} \epsilon_{ij} \left(\epsilon^{(n)j} + \epsilon^{(n+1)j} \right) \\
\delta \lambda^0 &= \dot{\epsilon}^0 \\
\delta \lambda^{(1)i} &= \frac{1}{\sqrt{\kappa}} \dot{\epsilon}^{(1)i} \qquad \delta \lambda^{(n+1)i} = \dot{\epsilon}^{(n+1)i} \\
\delta \lambda^3 &= \epsilon^0 + \dot{\epsilon}^3 \qquad n = 1, 2, \ldots
\end{aligned} \tag{27}$$

which are generated from the definition of gauge transformation generators as

$$G = \int \mathrm{d}^2x \left(\epsilon^0 \omega_0 + \epsilon^3 \omega_3 + \frac{1}{\sqrt{\kappa}} \epsilon^{(1)i} \tilde{\omega}_i^{(1)} + \sum_{n=2}^{\infty} \epsilon^{(n)i} \tilde{\omega}_i^{(n)} \right) \tag{28}$$

with infinitesimal gauge parameters ϵ^0, ϵ^3 and $\epsilon^{(n)i}$ $(n = 1, 2, \ldots, \infty)$ where we have inserted $(\kappa)^{-1/2}$ in front of $\epsilon^{(1)i}$ for convenience. The equations of motion of the Lagrange multipliers $\lambda^{(n)i}$ give $\tilde{\omega}_i^{(n)}$, while $\lambda^{(n)i}$ themselves can be gauged away. Using the gauge condition $\lambda^3 = 0$ and thus $\delta\lambda^3 = 0$ [13], equation (26) reduces to

$$S = \int \mathrm{d}^3x \left(\pi_0^{(0)} \dot{A}^{(0)0} + \sum_{n=0}^{\infty} \pi_i^{(n)} \dot{A}^{(n)i} - \lambda^0 \omega_0 + A^{(0)0} \omega_3 - \sum_{n=1}^{\infty} \lambda^{(n)i} \tilde{\omega}_i^{(n)} \right) \tag{29}$$

by identifying $\epsilon^0 = -\dot{\epsilon}^3$. Note that the partially gauge fixed action (29) is invariant under residual gauge transformations. To exhausted all additional gauge degrees of freedom, we choose gauge conditions $\chi^{(n)i} = \pi_i^{(n)} + (1/2)\epsilon_{ij} A^{(n)j} \approx 0$, $(n = 1, 2, \ldots)$ with equation (23) and $\lambda^{(n)i} = 0$, similar to the case of chiral boson [15]. We can, therefore, recover the original pure CS Lagrangian (1) maintaining only the usual U(1) gauge symmetry.

On the other hand, if one eliminates $\pi_0^{(0)}$, $\pi_i^{(0)}$, $\pi_i^{(n)}$, λ^0 and $\lambda^{(n)i}$ $(n = 1, 2, \ldots)$ from (29) by means of their own equations of motion, one could get once again the desired action (18) and compactly rewrite it as follows

$$\mathcal{L} = -\frac{\kappa}{2}\epsilon_{ij}\left(A^{(0)i} + \frac{1}{\sqrt{\kappa}}\sum_{n=1}^{\infty}A^{(n)i}\right)\left(\dot{A}^{(0)j} + \frac{1}{\sqrt{\kappa}}\sum_{n=1}^{\infty}\dot{A}^{(n)j}\right) + \kappa A^{(0)0}\epsilon_{ij}\partial^i\left(A^{(0)j} + \frac{1}{\sqrt{\kappa}}\sum_{n=1}^{\infty}A^{(n)j}\right). \qquad (30)$$

Then one can easily check that this action is invariant under the following local gauge transformations

$$\delta A^{(0)0} = \partial^0\Lambda \qquad \delta A^{(0)i} = \partial^i\Lambda + \frac{1}{\sqrt{\kappa}}\epsilon^{(1)i} \qquad \delta A^{(n)i} = -\epsilon^{(n)i} + \epsilon^{(n+1)i} \qquad n = 1, 2, \ldots \qquad (31)$$

where we simply define $\epsilon^3 = -\Lambda$. The transformation rules show the usual U(1) gauge transformation with the gauge parameter Λ and a new type of local symmetry with $\epsilon^{(n)i}$.

It seems to be appropriate to comment on the Lorentz covariance of equation (30). In BFT formalism, the non-covariance of the resulting action is mainly due to the non-covariant property of the second-class constraints ω_i in (6). To recover the Lorentz covariance of the action, we should introduce an additional infinite number of auxiliary fields as $A^{(0)0} \to A^{(0)0} + \sum_{n=1}^{\infty} A^{(n)0}$. Then (30) could be covariantly written by the covariant transformation rule. This including the non-Abelian extension has been studied in detail in [17].

In conclusion, we have found a new type of WZ action for the Abelian pure CS theory. To make two initial second-class constraints originating from the symplectic structure into a first-class system, we have introduced an infinite number of auxiliary fields via BFT formalism. It is remarkable that not only is the original U(1) gauge symmetry preserved but there also exist additional novel symmetries. Further, the derived WZ action is eventually independent of field theoretic models which involve the CS term.

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